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# SHARPENING OF INPUT EXCITATION CURVES IN LATERAL INHIBITION

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Various lateral inhibition neural networks are studied by means of computer simulation regarding their ability to sharpen the input excitation curves. To quantify this ability we introduced two new entropy-like quantities (called the iteration entropy and the rate of convergence) which represent suitable measures for the sharpening of the input excitation curves in a certain lateral inhibition neural network. Using these quantities we quantitatively described the sharpening ability of different lateral inhibition networks.

## 1. Introduction

The lateral inhibition is a well-known phenomenon in neurophysiology.[12] It has been found at a number of places in CNS, e.g. in nucleus cuneatus and thalamus as a part of the somestetic afferentation.[10] Early studies on lateral inhibition in different neural networks appeared in the sixties.[11,8] They were motivated by the possible existence of the lateral inhibition mechanism in some sensory analysers.[4,3] Since then the lateral inhibition networks are the subject of continuous interest[1] mainly for their ability of sharpening the input excitation curves in sensory analysers. As is well known the frequency discrimination tuning curves in cochlea[5] are too broad for the fine frequency discrimination of the human ear, therefore a sharpening procedure must take place somewhere in neural network of auditory system. A neural network with the lateral inhibition seems to be most suitable for this sharpening procedure. Lateral inhibition is also connected with the problems of contrast enhancement and with mechanisms that prevent the interference of different neural processes. These networks can be used as maximum-finders, e.g. the Hamming network.[7] The question arises: How are various types of lateral inhibition networks effective in sharpening of different input excitation curves? In order to answer this question we studied several lateral inhibition network models and quantify them with respect to this ability.

## 2. The Model

We consider neural networks of $L$ one-dimensional layers each of which consists of $N$ neurons (processing elements–PEs). The neurons of the $x$th layer are only connected with neurons of $(x+1)$th layer. Denoting the activity (pulsation) of $i$-neuron in $x$th layer as $\varphi[i]_x$, the activity of $i$th neuron in $(x+1)$th layer is given by the equation

$$\varphi[i]_{x+1} = f\left(\sum_{i-M}^{i+M} w[k]\varphi[k]_x - \Theta_k\right), \qquad (1)$$

where $f$ is the step function

$$f(t) = \begin{cases} 1 & \text{if} \quad t > 0 \\ 0 & \text{if} \quad t \le 0 \,. \end{cases}$$

$w[i]$ is the corresponding connection weights and $\Theta$ is the neuron threshold value. $w[i]$ is positive or negative for the excitatory or inhibitory connection of the neighbouring neurons, respectively. There are different types of one-dimensional networks according to the weights one takes in Eq. (1). If $w[i]$ is always positive (i.e. the connection between $i$th neuron in

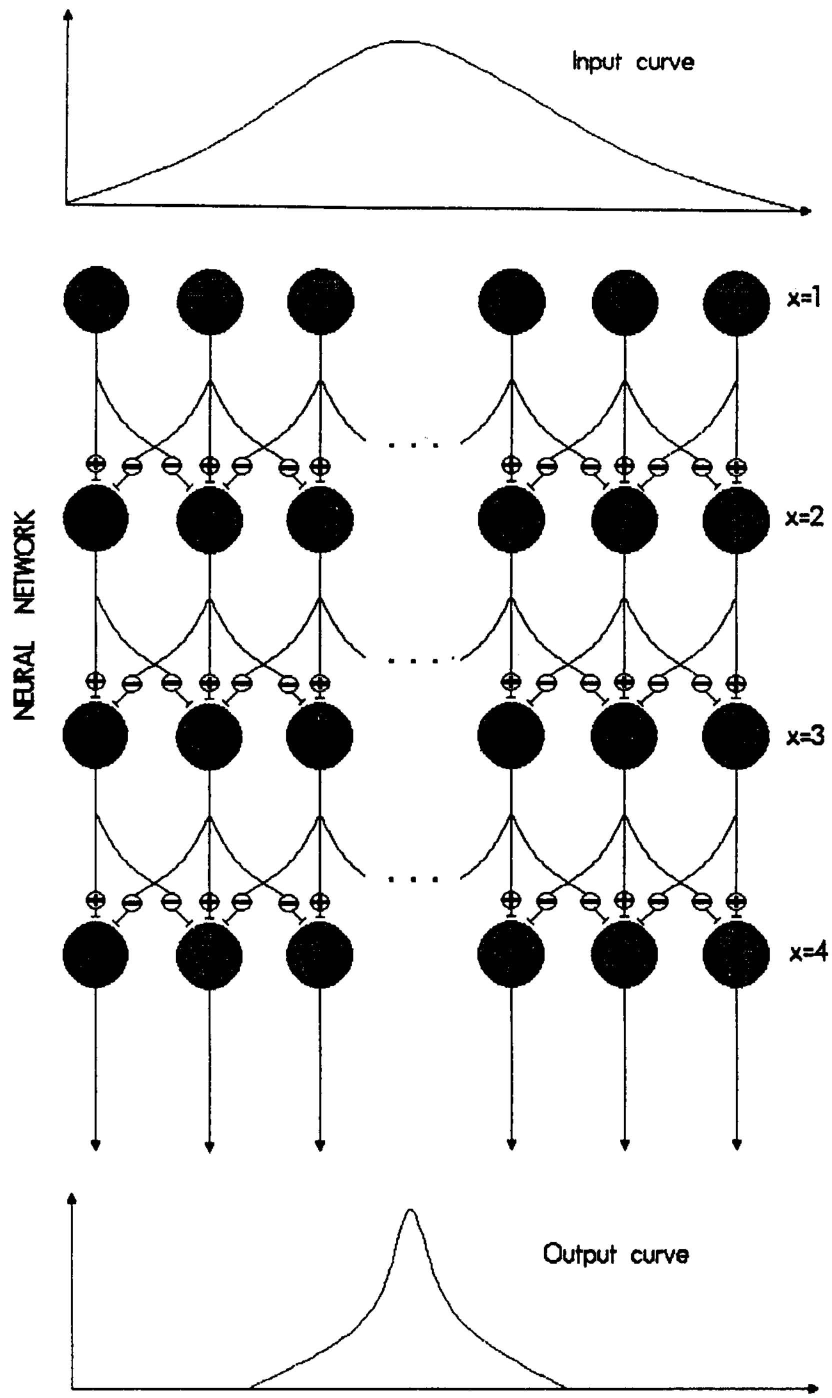


Fig. 1(a). Architecture of a lateral inhibition network.

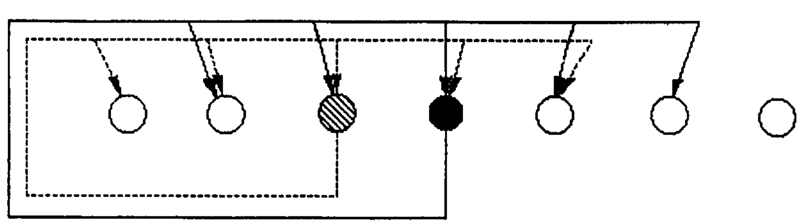

Fig. 1(b). Interneural connections.

$x$th layer and $i$th neuron in $(x+1)$th layer is an excitatory one) and $w[k]$'s ($k = i-M,\ i-M+1,\ i-M+2, \ldots, i-1,\ i+1, \ldots, i+M$) (i.e. the connection between $k$th neuron in $x$th layer and the $i$th neuron in $(x+1)$th layer) is an inhibitory one, then we have a lateral inhibition network (see Fig. 1(a)). Each one-dimensional lateral inhibition neural network (LINN) is given by the following parameters:

(i) the excitatory weight $w[i]$,
(ii) the inhibitory weights $w[k]$,
(iii) the threshold value $\Theta$ and
(iv) the connectivity defined as $c = M/N$.

According to the choice of the foregoing parameters, there is a variety of possible LINNs. In the computer modelling the real one-dimensional $L$-layered neuron network is represented by an iterative network (i.e. the neurons' outputs are led back to the input of the network). Here, the $x$th iteration in computer model represents the state of $x$th neuron layer of the real network.

In order to quantify the different LINNs regarding their sharpening ability we introduce a measure for it which has the form of entropy of a probability distribution. It is well known that the steeper the probability distribution is, the smaller is the value of its entropy. This property can be also used for the measure of the sharpening ability in a lateral inhibition network. Let $V_x$ be the normalized activation vector of $x$th layer ($x$th iteration) defined as

$$V_x \equiv (\bar{\varphi}[1]_x,\ \bar{\varphi}[2]_x, \ldots, \bar{\varphi}[N]_x)\,,$$

where

$$\bar{\varphi}[k]_x = \frac{\varphi[k]_x}{\sum_{i=1}^{N} \varphi[i]_x}\,.$$

We introduce the quantity (in what follows we will call it as iteration entropy)

$$E(x) = -\sum_{i=1}^{N} \bar{\varphi}[i]_x \ln(\bar{\varphi}[i]_x)$$

and define the measure of sharpening ability (called the rate of convergence) of a network in $x$th iteration (layer) as the difference of $E(x-1)$ and $E(x)$

$$R(x) = E(x-1) - E(x)\,.$$

The larger the $R(x)$ is, the more the processed curve becomes sharper in $x$th iteration. It can also be expected that the sharper the output curve (i.e. the curve in the output $L$th layer) is, the lower

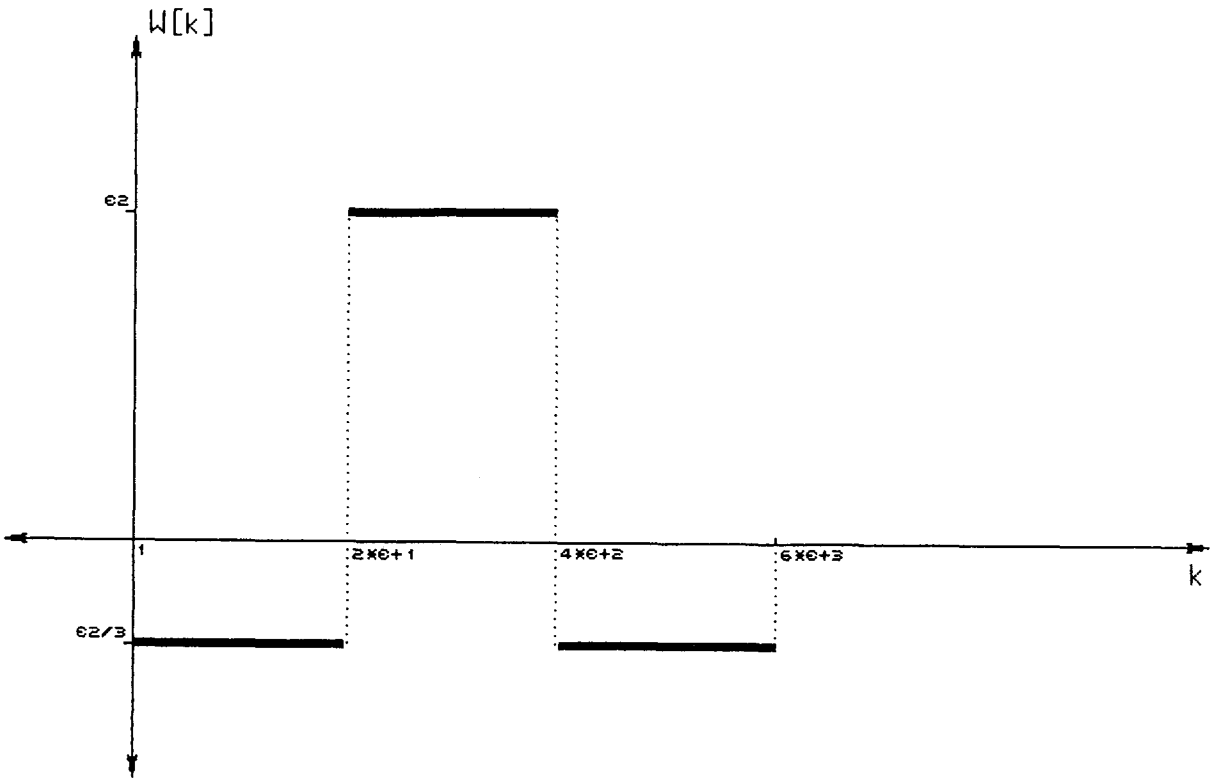


Fig. 2. Weights in Kohonen's network.

value gets the corresponding $E(L)$. If $E(L)$ is zero, then there is only one neuron firing in the output network layer. Such network can be used as an ideal maximum finder. We see that $E(x)$ is in a sense a measure of the amount of information on the maximum of the processed curve in a LINN. In what follows we investigate three actual models of LINN with respect to their ability of sharpening the different input curves.

## 3. Various Networks

### 3.1. *Kohonen's network*

Let us first consider the network published by Kohonen.[6] The neural connections of this network as well as the corresponding weights are shown in Fig. 1(b) and 2. Note that in this network

$$M = 6\varepsilon + 3\,.$$

The constants $\varepsilon$ and $\varepsilon 2$ in Fig. 2 had to be adjusted according to the selected connectivity and necessary stability during the iterations. Taking $\varphi[i]_0$ as the external input excitation of $i$th neuron, the activation function of $x$th layer has the form

$$a[i]_x = \sum_{k=1}^{M}(w[k]\cdot\varphi[i-(M \text{ div } 2) + k - 1]_{x-1}) + \varphi[i]_0 - \Theta\,, \qquad (2)$$

where $\Theta$ is the threshold of the neuron (we take all the thresholds equal) selected experimentally according to the *signal-noise ratio*.

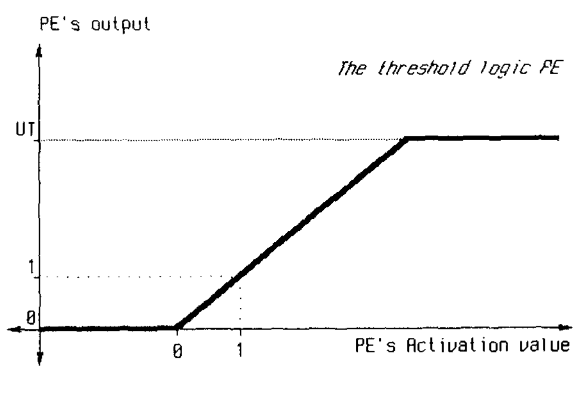


Fig. 3. The output function of Kohonen's network ($f_T$).

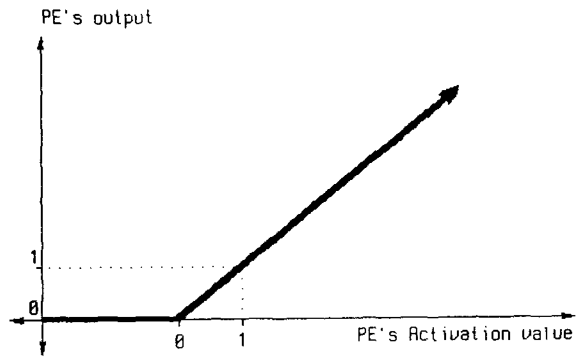


Fig. 4. The output function of Hamming network ($f_i$).

The threshold logic function (Fig. 3) was used to compute the output:

$$\varphi[i]_x = f_T(a[i]_x)\,.$$

### 3.2. *Hamming network (maxnet of Hamming net[7])*

The anatomy of the network is shown in Fig. 1(b). The weights are adjusted by the following way:

$$\forall k \varepsilon \{1,\ldots,(M \text{ div } 2)\} \cup \{(M \text{ div } 2)+2,\ldots,M\}: \quad W[k] = \varepsilon = -\frac{1}{M}$$

$$k = (M \text{ div } 2) + 1: \quad W[k] = 1\,.$$

The activation function is

$$a[i]_x = \sum_{k=1}^{M}(w[k]\cdot\varphi[i-(M \text{ div } 2) - 1 + k]_{x-1}) \quad (3)$$

and the output value is determined by the identity function (Fig. 4)

$$\varphi[i]_x = f_i(a[i]_x)\,.$$

### 3.3. *LINN-1*

Our model (in what follows we denote it as LINN-1) is based on the same equations as Hamming net except a nonzero threshold (its aim is described below) and the fact that a kind of normalisation was performed after each iteration:

$$\varphi[i]_x = \frac{\varphi[i]_x}{\max\{\varphi[1]_x, \varphi[2]_x, \ldots, \varphi[N]_x\}}\,. \qquad (3a)$$

This normalisation is due to ensuring the stability of the signal processing during the iterations. That is necessary because, e.g. in Hamming net, the signal dies-out if there is no clear peak in the signal input.

## 4. Boundary Fixing

Finally, there is a problem connected with the anatomy of the network due to the limited length of the layers. The neurons on the boundary of a layer have no neighbours and Eq. (1) cannot be applied here. We treated this problem by the following way. The boundary neurons on one side are connected with the boundary neurons on the other side so that a layer forms a one-dimensional ring.

## 5. The Input Curves

We used the following input curves ($\varphi[i]_0$) in the experiments described below:

(1) *Single-hump curves* given by the formula

$$\varphi[i] = \frac{q}{1 + \dfrac{(i-l)^n}{p}} \quad (l,\ p,\ q \text{ are constants}).$$

We have chosen the following values for the variables $q$ and $n$ : $q = 1$; $l = 50$; $p = 50$ and $n = 2$ (curve $G - 2$); $n = 4$ (curve $G - 4$); $n = 8$ (curve $G-8$); $n = 0.5$ (curve $G$-SQRT) and $n = 0.25$ (curve $G$-SQRT2).

(2) *Sinusoid curves* defined as

$$\varphi[i] = \text{ABS}\left[\text{SIN}\left(\frac{m \cdot i}{N} \cdot \pi\right)\right]^n .$$

We have taken the following values for the exponent $n$: 2, 4, 8, 0.5 and 0.25. This parameter determines the steepness of the curve. For $m$ which determines the number of maximas, we have taken the values 1, 2 and 4.

(3) *The step curve* BREAK the form of which is shown in Fig. 8.

## 6. Results

In what follows we present the relevant results of the computer simulation of the various LINNs with different input curves. In the first part we show the actual form of the iterated curve of the network's output in a certain iteration. Then we present the iteration entropy as a function of the number of iterations $x$. Finally, we treat the measure of sharpening ability (rate of convergence) in a certain iteration $R_x = E(x) - E(x-1)$ as a function of the number of iterations.

The form of the iterated curves (the outputs of the network) for the input curve $G-2$ in LINN-1 with $\Theta = 0$ is already deformed in the second iteration

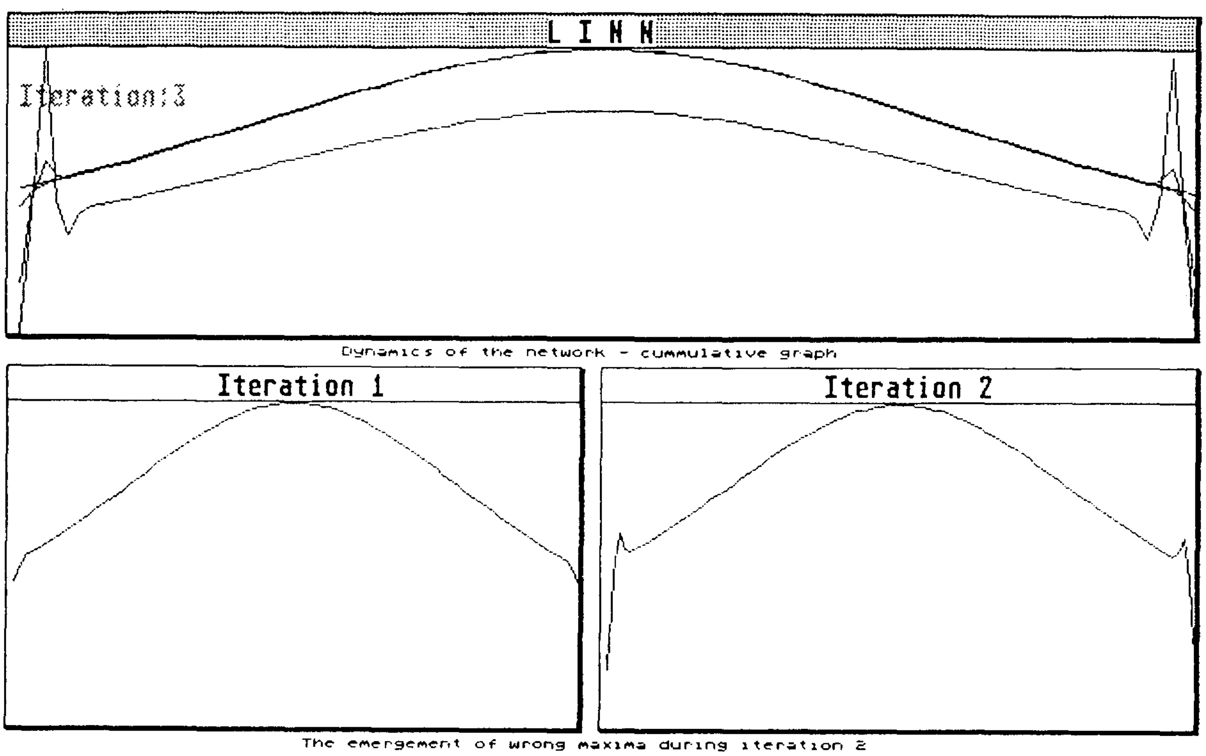


Fig. 5. Parameters: $M = 5$; $\Theta = 0$; $\varepsilon = -0.2$; $N = 100$. Curve: $G - 2$.

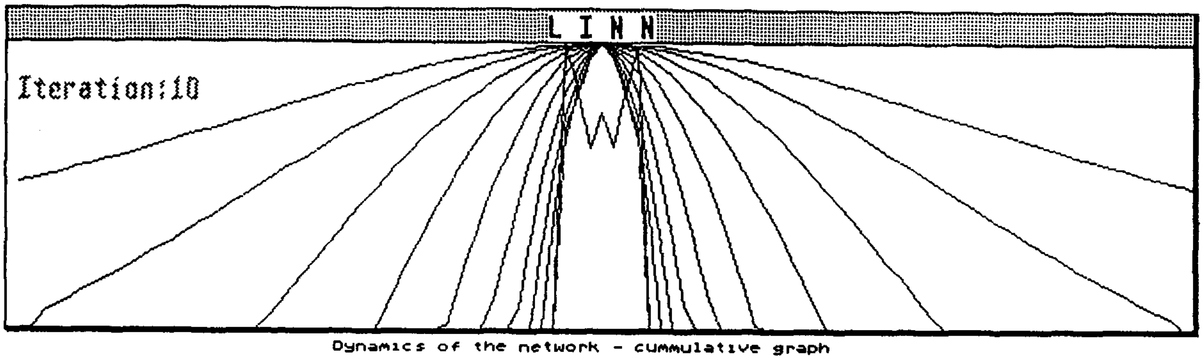


Fig. 6. Parameters: $M = 5$; $\Theta = 0.1$; $\varepsilon = -0.2$; $N = 100$. Curve: $G - 2$.

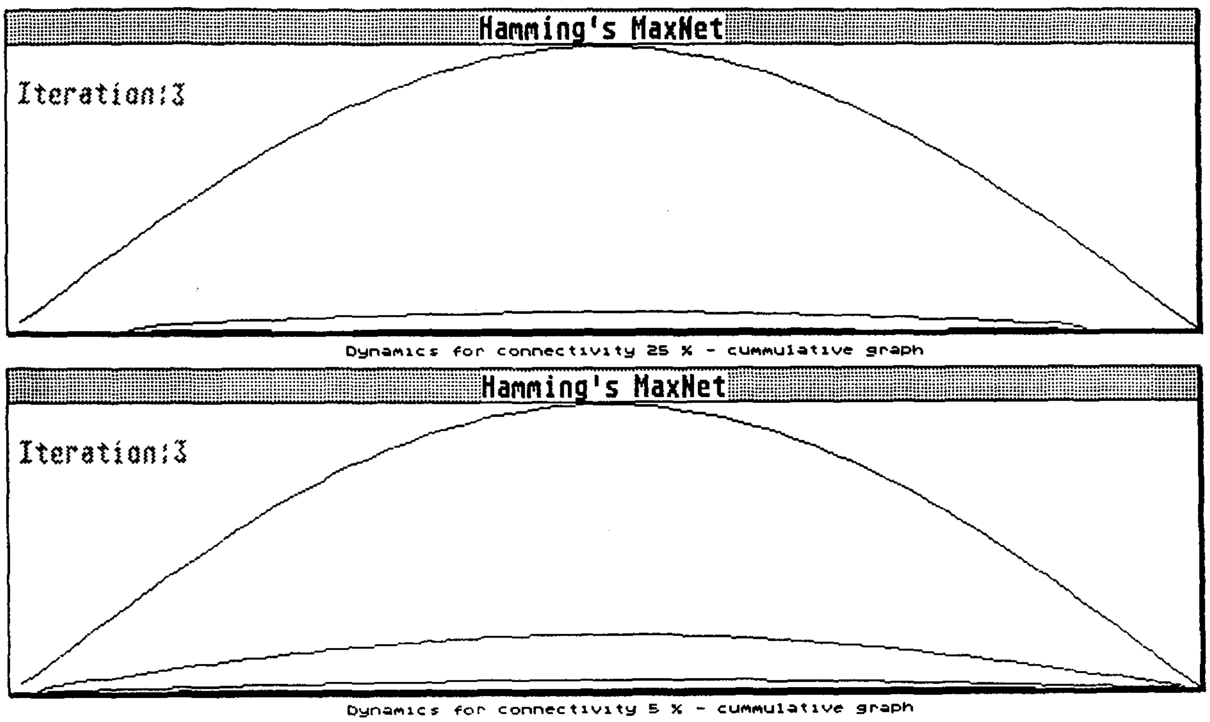


Fig. 7. Parameters: $M = 100$; $\Theta = 0$; $\varepsilon = -0.01$; $N = 100$. Curve: sinusoid type with $n = m = 1$.

(Fig. 5) whereas those in LINN-1 with $\Theta = 0.1$ are deformed first in the 10th iteration (Fig. 6). Here we see also that the individual iteration curves have the sharpened form of the input curve. The dying-out of the iterated curves in Hamming net is demonstrated in Fig. 7. The maximum of the iterated curves becomes smaller quickly during the iteration process. We see that the dying-out of the iterated curves is larger at greater connectivity. The deformation of the iterated curves for the input curve "BREAK" and the sinusoid curve in Kohonen's net is shown in Figs. 8 and 9. Whereas the curve "BREAK" is relatively more deformed, sinusoid curves are only sharpened but not deformed. Here we can conclude that the deformation of iterated curves is smaller, the smoother the input curves are.

Next we quantitatively evaluate the sharpening procedure in various LINNs by means of the iteration entropy $E(x)$ and the rate of convergence $R_x = E(x) - E(x-1)$. We study both quantities as a function of the number of iterations. The function $E = f(x)$ tells us how the absolute value of $E$ decreases with the number of iterations and $R = f(x)$ tells us how effective it is in each individual iteration. If $E(x)$ gets zero then only one neuron is active and the sharpening procedure is finished. This can be seen, e.g. in Fig. 10 where the dependence of $E$ on $x$ with input curve $G$-SQRT and for different

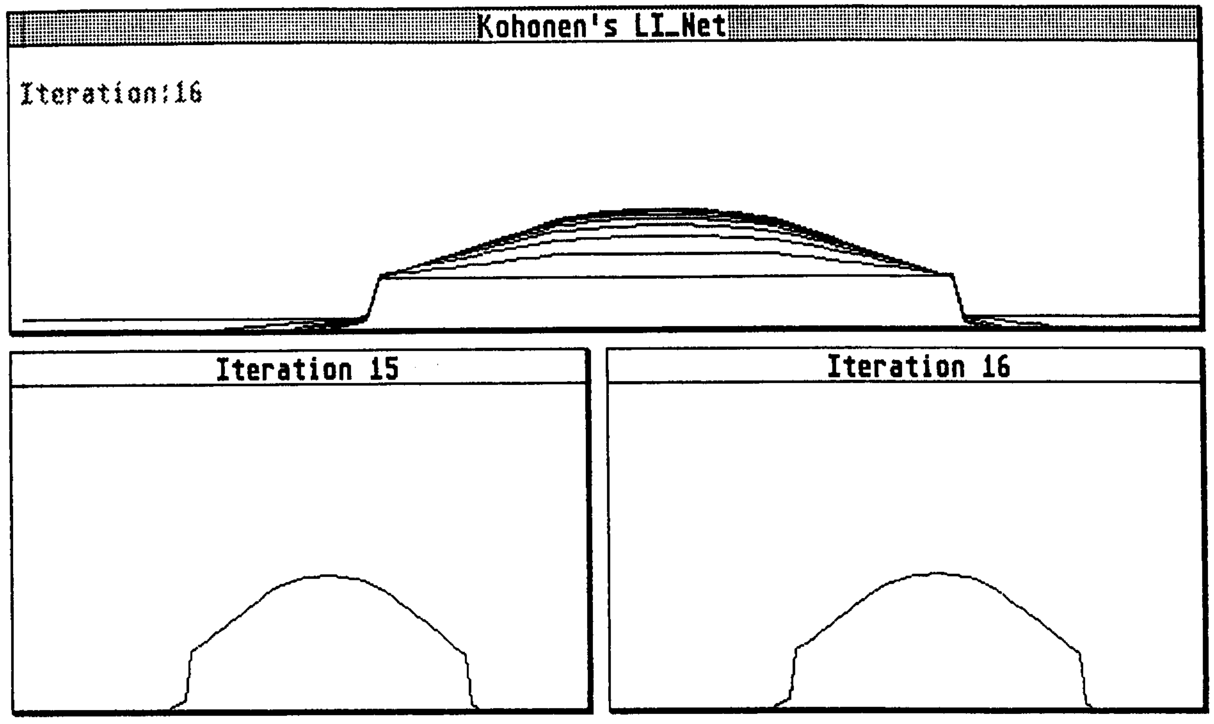


Fig. 8. Parameters: $M = 93$; $\Theta = 0.1$; $\varepsilon = 15$; $\varepsilon 2 = 0.025$; $N = 100$. Curve: BREAK.

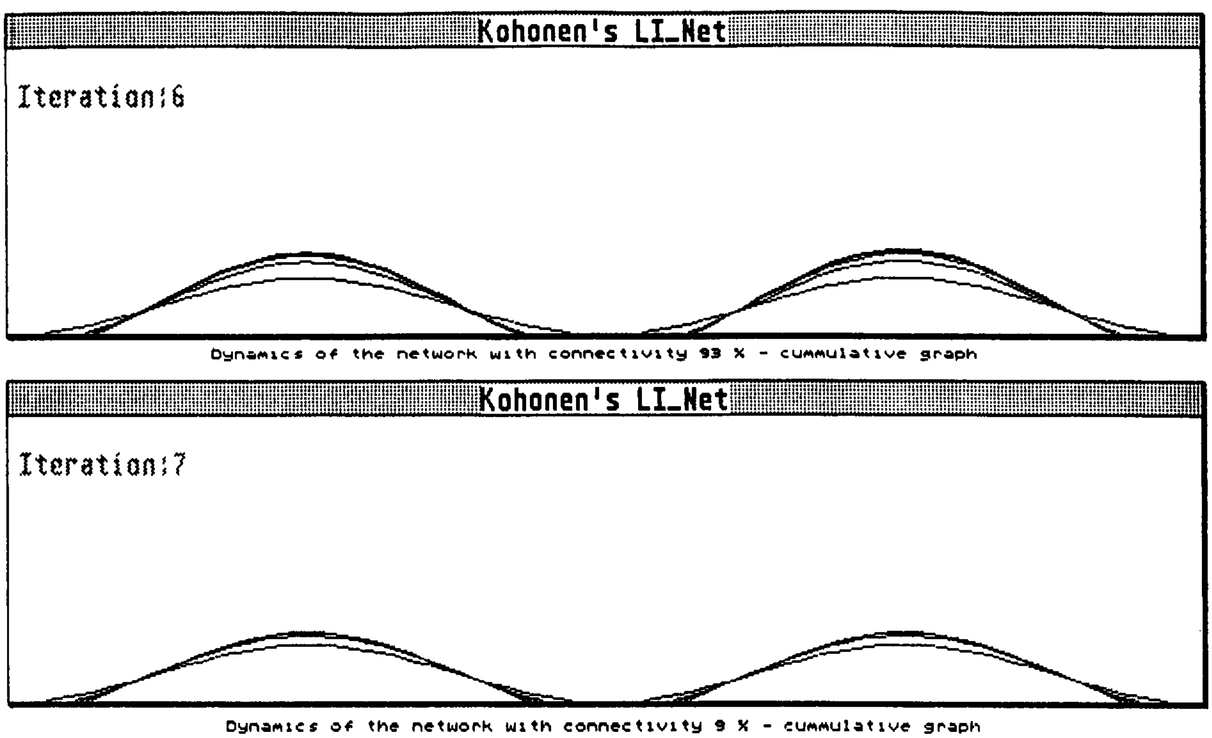


Fig. 9. Parameters: $\Theta = 0.1$; $N = 100$. Top: $\varepsilon = 15$; $\varepsilon 2 = 0.025$; $M = 93$. Bottom: $\varepsilon = 1$; $\varepsilon 2 = 0.25$; $M = 9$. Curve: Sinusoid type, $n = m = 2$.

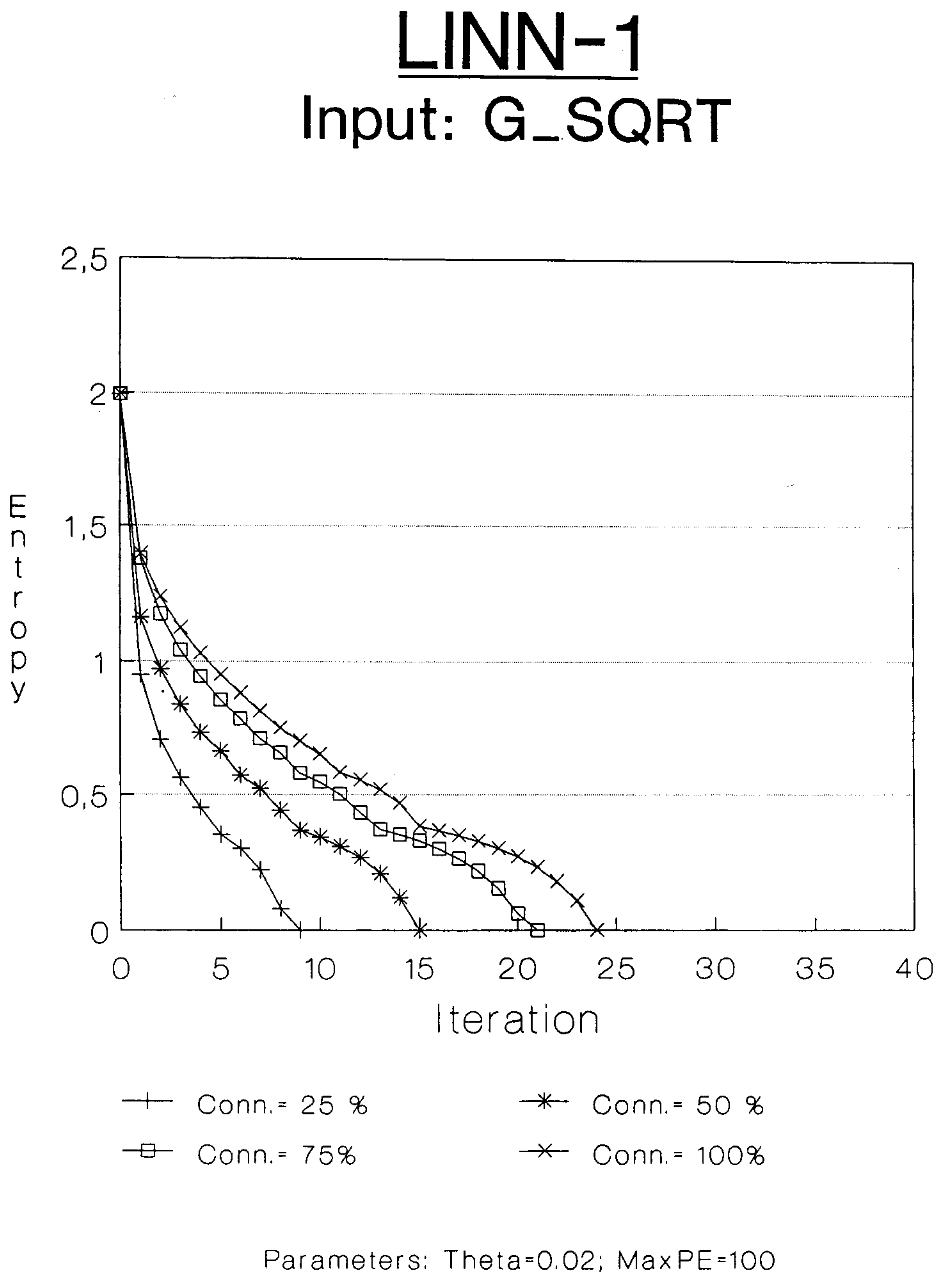


**Fig. 10.** ***E* as a function of the number of iterations for LINN-1.**

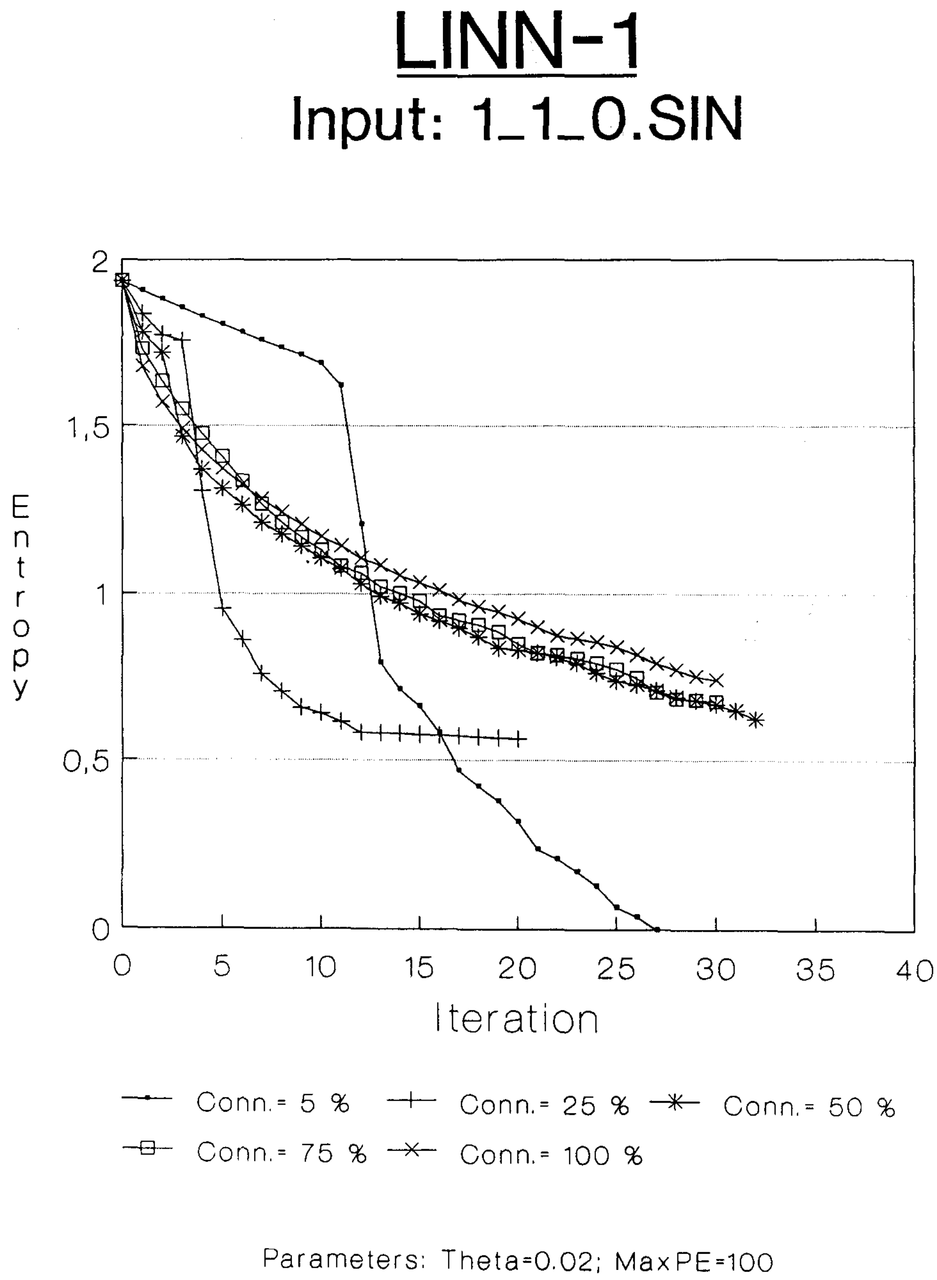


**Fig. 11.** *E* **as a function of the number of iterations for LINN-1 with a sinusoid input signal.**

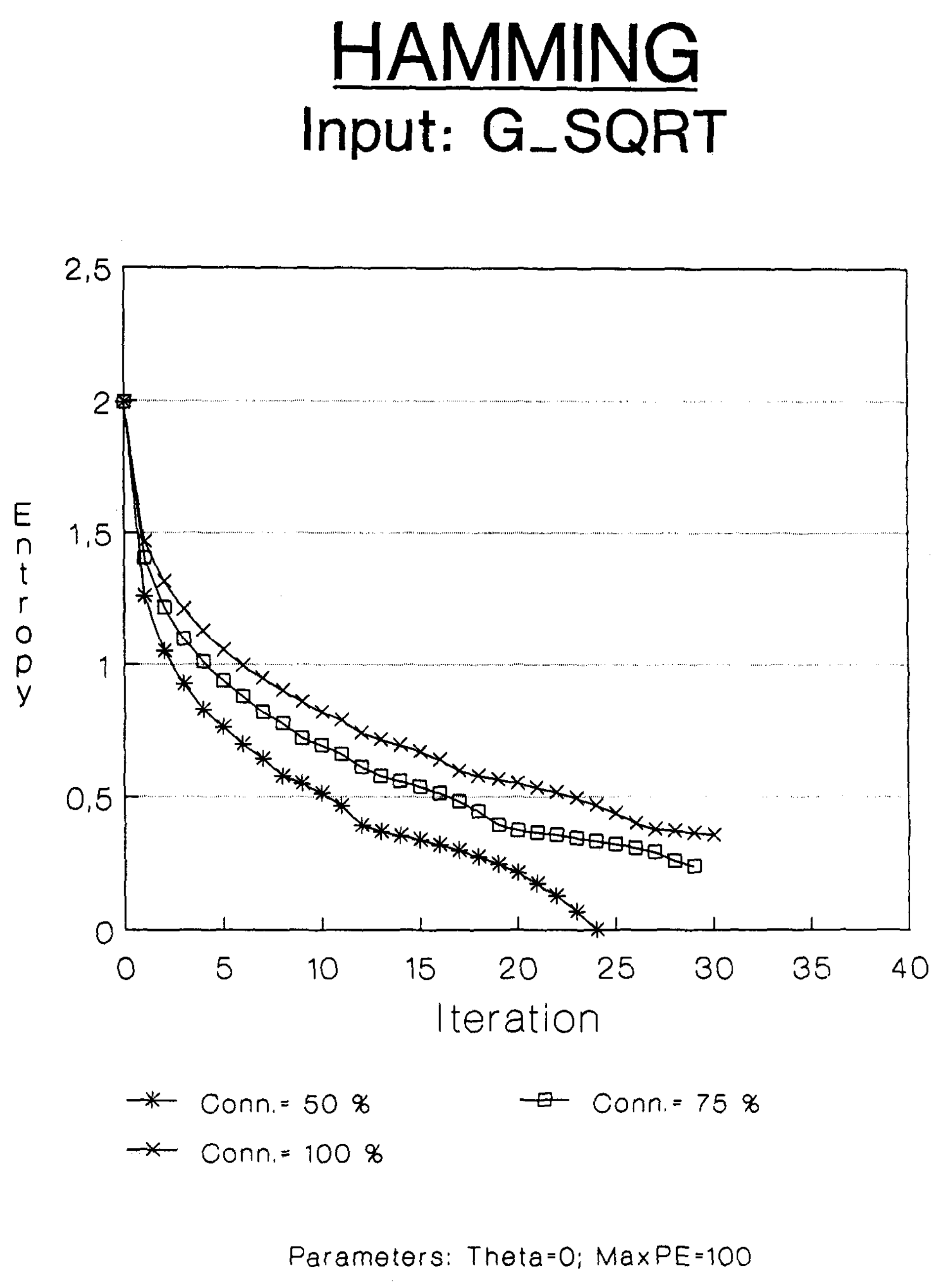


Fig. 12. $E$ as a function of the number of iterations for Hamming net.

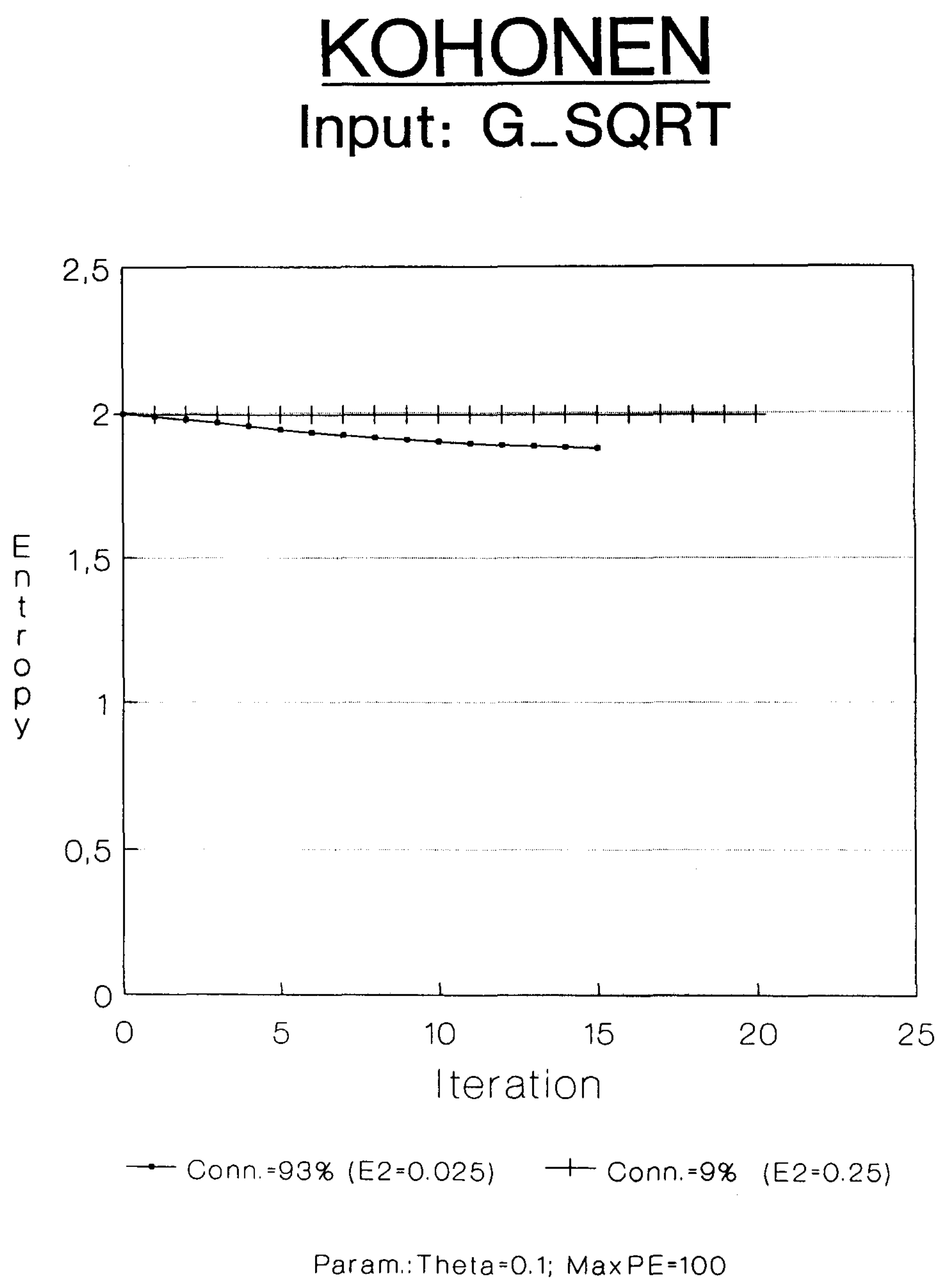


**Fig. 13.** ***E*** **as a function of the number of iterations for Kohonen's net.**

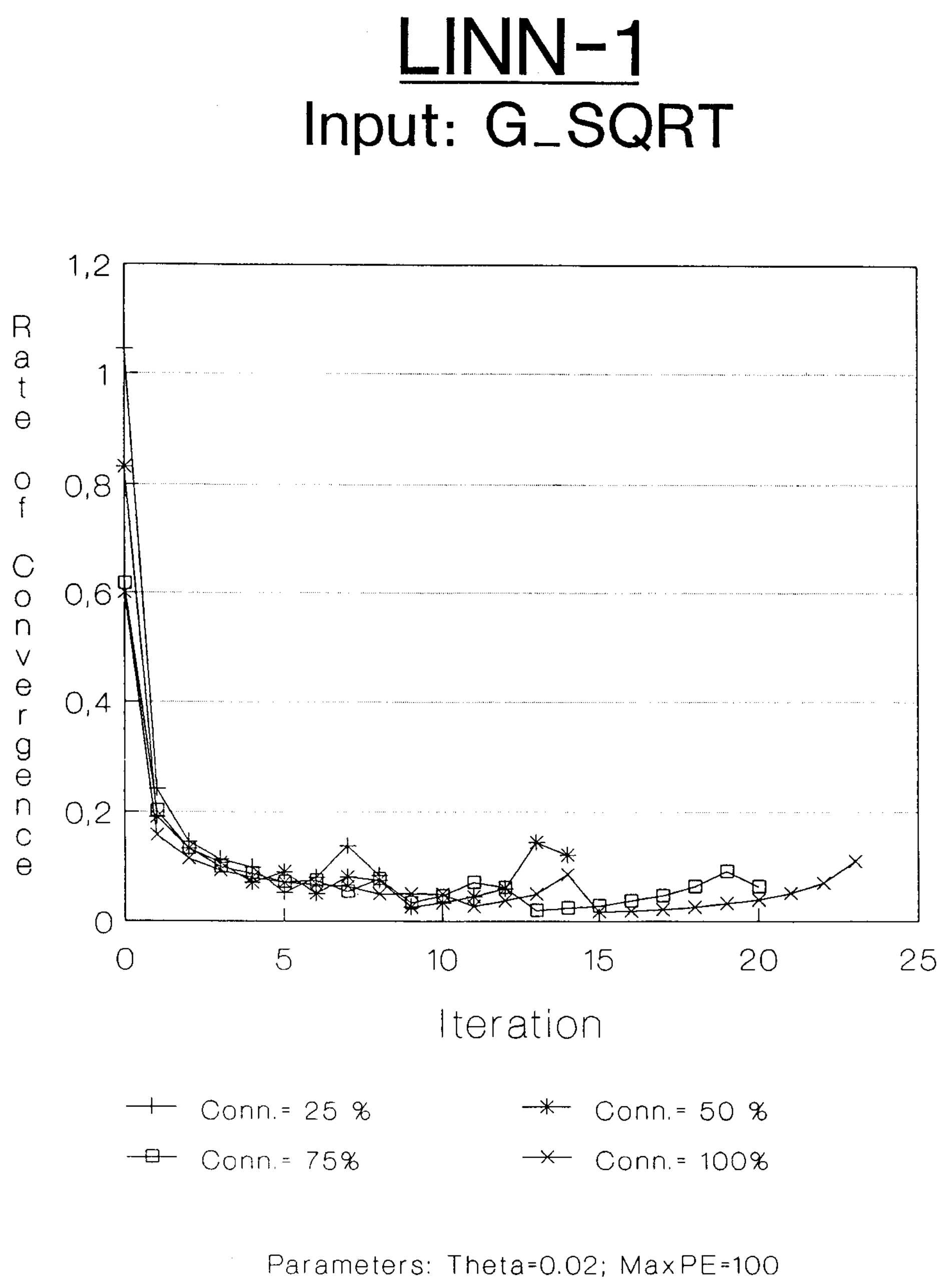


**Fig. 14.** ***R*** **as a function of the number of iterations for LINN-1.**

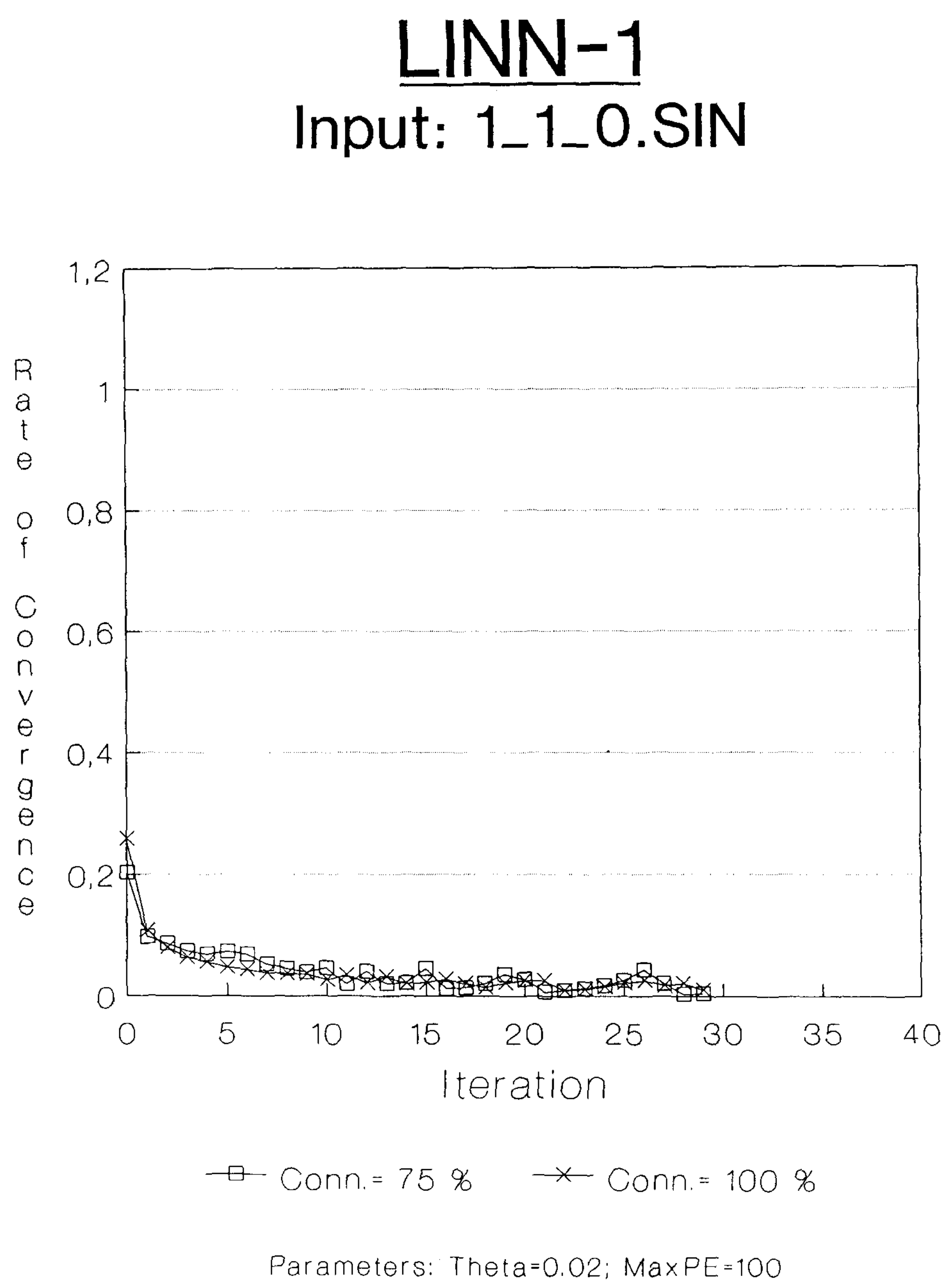


**Fig. 15.** *R* as a function of the number of iterations for LINN-1 with a sinusoid input signal.

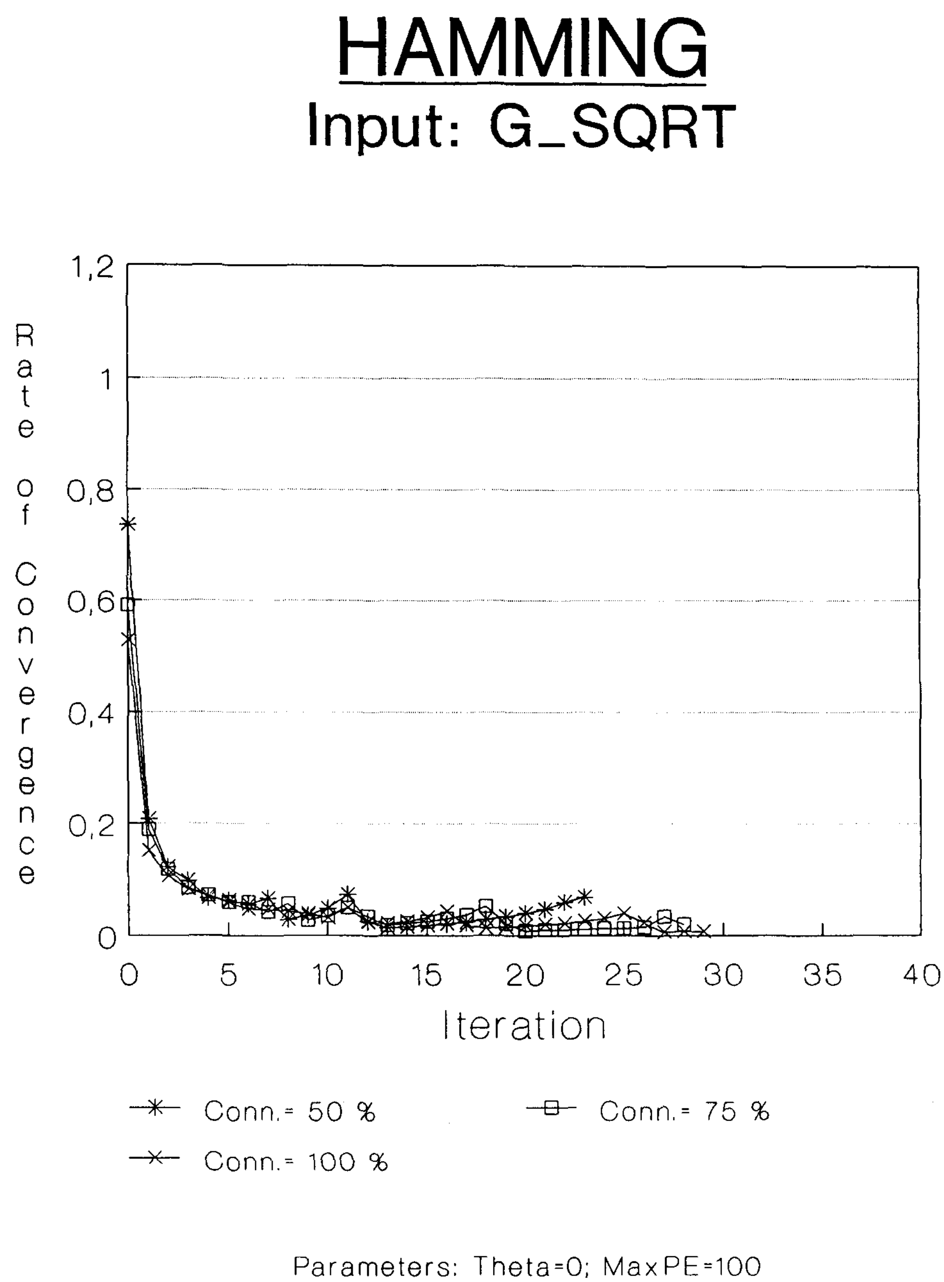


**Fig. 16.** ***R*** **as a function of the number of iterations for Hamming net.**

values of connectivity is depicted. We conclude that the larger the connectivity is, the longer is the sharpening procedure. We see also in Fig. 11, where $E$ is shown as a function of the number of iterations for the input curve 1–1–0.SIN, that here, except for connectivity 5%, no values of $E$ are zero even after the 30th iteration. There is the similar situation also in Hamming net (Fig. 12) where $E$ gets to the zero level only for connectivity 50%. $E = f(x)$ for Kohonen net with input curve $G$-SQRT is shown in Fig. 13. We see that the decrease of $E$ during the iteration procedure is very slow for connectivity 93% and almost constant for connectivity 9%.

The magnitude of the sharpening in each individual iteration can be best demonstrated by means of the rate of convergence $R(x)$.

In Figs. 14, 15 and 16 we see that the rate of convergence as a function of the number of iterations exhibits the following features: in interval 0–5 iterations it is considerably decreasing and in interval 5–25 it is practically constant. That indicates that the ***first five iterations*** are the most effective with respect to the sharpening ability of the individual types of LINNs. The connectivity as a parameter does not play a very important role in sharpening ability in the interval of 5–30 iterations. The input curve seems to be more important than the connectivity. We see it by the comparison of Figs. 14 and 15. The rate of convergence for the input curve $G$-SQRT is approximately twice as large as that for the curve 1–1–0.SIN in the interval of 5–30 iterations. Hamming net has the typical form of the function $R = f(x)$ as well. The largest value of the rate of convergence has the LINN-1 for the input curve $G$-SQRT.

## 7. Discussion

*Kohonen's network* is the only network that can assure stability and error-free iteration simultaneously. Possibly it can serve as a mechanism for **preserving the potential maps** over natural neural networks. The slight modifications of the iteration curves in this network are aimed either at **contour enhancement** (those with low connectivity, Fig. 17) or at **smoothing** of the input (high connectivity, Fig. 8). This net can also act like a **filter** emphasizing some characteristics of the input signal.

*LINN-1* is a network where a **superviser** has to be present to choose the maximum for the normalisation.

A shortcoming of the *Hamming net* is the rapid **fading-out** of activity especially with input curves without clear peak. This can be prevented either through changes in weights (the excitatory weight selected $> 1$) or through adding the input to the iteration formulas (see above). But then we are

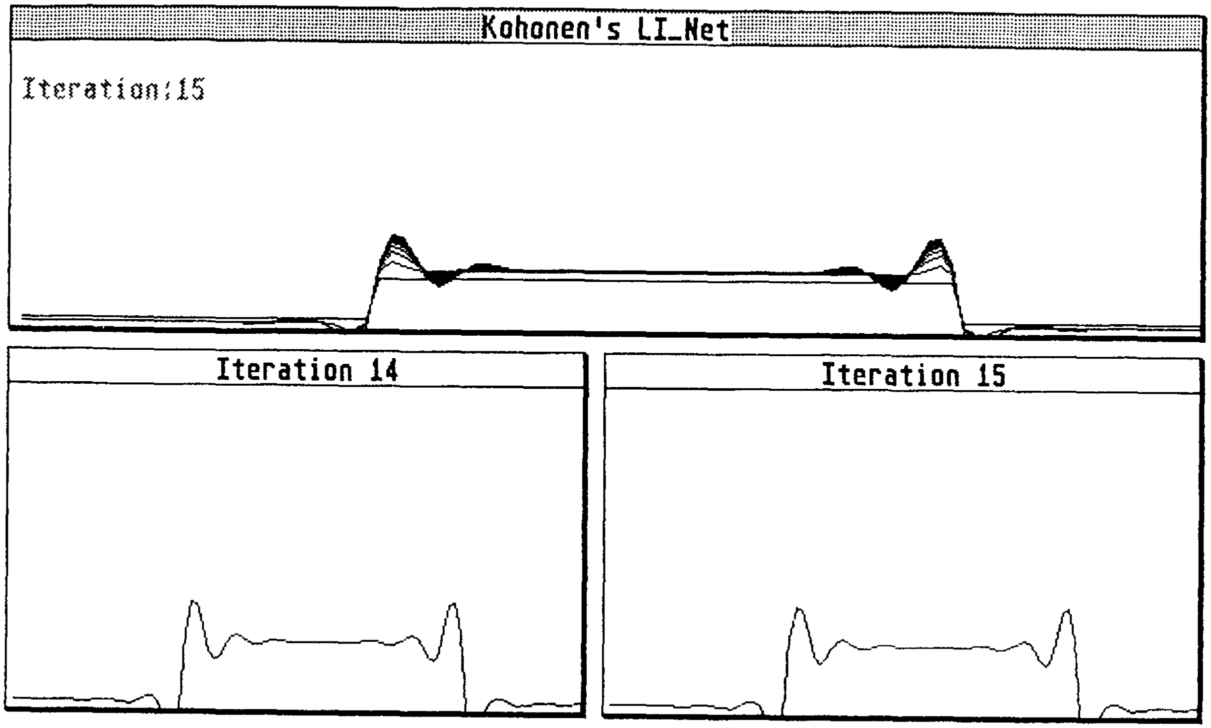


Fig. 17. Parameters: $M = 9$; $\Theta = 0.1$; $\varepsilon = 1$; $\varepsilon 2 = 0.25$; $N = 100$. Curve: BREAK.

confronted with the wrong-maxima problem. So these networks (due to the way how wrong maxima develop ) may not be used as iterative networks but they fit as networks for just one passing where **contour-enhancement** is necessary. Where a **maximum-finder** is asked (the connectivity 100%) we may use them even in the **iterative mode**; here no wrong maxima evolve.

Let us finally draw some simple implications for the real neural phenomena. Due to the fact that only the first 5 iterations are the most effective for sharpening, it does not seem necessary to have iterative LINNs in the architecture of CNS. This is in accord with the so-called **100 step program constraint**[2] which arose due to the fact that neurons operate in the time-scale of milliseconds and so neural processes that take 1 second can involve only a hundred or so steps.[9]

## References

1. M. Cottrell, "Mathematical analysis of a neural network with inhibitory coupling," *Stoch. Proc. and their Appl.* **40**, 103–126 (1992).
2. J. A. Feldman, "Connectionist models and their applications: Introduction," *Cognitive Sci.* **9**, 1–2 (1985).
3. G. G. Fruman and L. S. Frischkopf, "Model of neural inhibition in mammalian cochlea," *J. Acoust. Soc. Am.* **36**, 2194 (1964).
4. H. K. Hartline, "Receptor mechanism and the integration of sensory information in the eye," *Rev. Mod. Phys.* **31**, 515 (1959).
5. M. H. Holmes, "Frequency discrimination in mammalian cochlea: Theory versus experiment," *J. Acoust. Soc. Am.* **81** (1987).
6. T. Kohonen, "Self-organized formation of topologically correct feature maps," *Biol. Cybern.* **43**, 59–69 (1982).
7. R. P. Lippmann, "An introduction to computing with neural nets," *IEEE ASSP Mag.* **4**, 4–22 (1987).
8. V. Majernik, "Über Neuronennetze als Modelle der Frequenzanalyse des Gehörs," *EIK* **3**, 115–125 (1967).
9. J. McClelland and D. Rumelhart, *Parallel Distributed Processing* (The MIT Press, England, 1989).
10. K. Popper and J. Eccles, *The Self and Its Brain–An Argument for Interactionism* (Springer Verlag Heidelbarg, New York, 1977).
11. W. Reichardt and G. MacGintie, "Zur Theorie der lateralen Inhibition," *Kybernetik* **1** (1961–1963).
12. R. F. Schmidt, *Fundamentals of Neurophysiology* (Springer Verlag, Tokyo, 1985).